2024-2025
Fall Semester

Course of Power System Analysis

Short-Circuit Current Calculation

Prof. Mario Paolone
Distributed Electrical Systems Laboratory
École Polytechnique Fédérale de Lausanne (Switzerland)

Outline

Study of symmetrical components of a symmetrical three-phase system with a generic triplet of impedances to ground

Three-phase short-circuit

Ungrounded two-phase short-circuit

Two-phase short-circuit with connection to ground

Single-phase short-circuit to ground

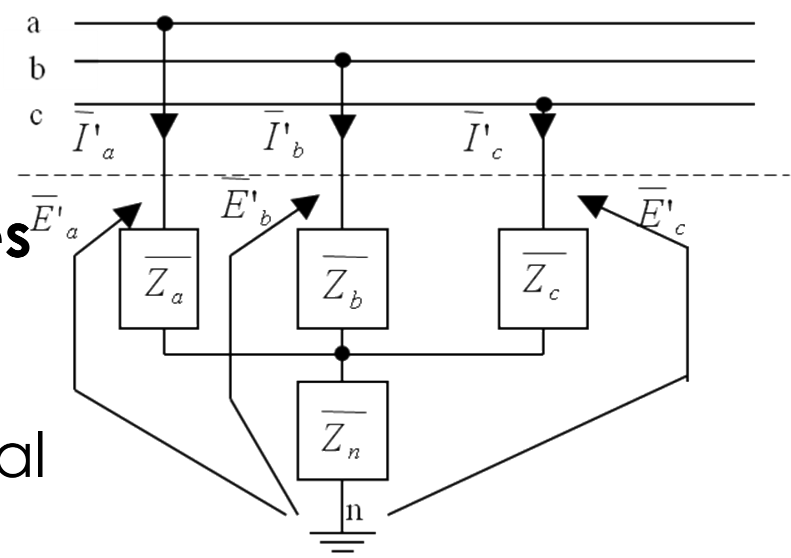
Study of symmetrical components of a symmetrical three-phase system with a generic triplet of impedances to ground

This part describes the approach necessary to study the **behaviour of an electric circuit in the event of a short-circuit (three-phase, two-phase, single-phase, etc.)** at any point in the network. A short-circuit in the network involves the connection of a **generic triplet of impedances to ground** and therefore an **asymmetrical and unbalanced regime.**

Study of symmetrical components of a symmetrical three-phase system with a generic triplet of impedances to ground

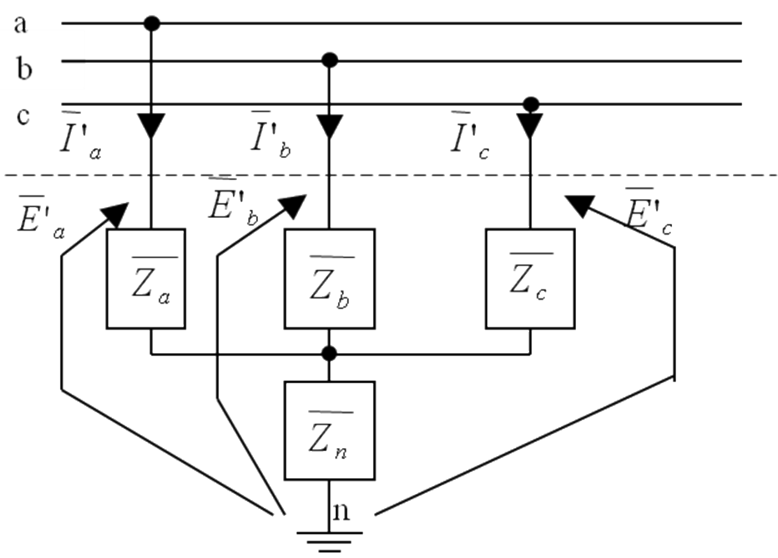
Hypotheses:

- electric networks made up of symmetrical elements (i.e., represented by circulant impedance matrices);
- an **asymmetrical set of impedances is connected at a generic point** in the network (see figure);
- at the point where the asymmetrical system of impedances is connected, the triplet of phase voltages across the impedances is $\bar{E}'_a, \bar{E}'_b, \bar{E}'_c$ and the trio of currents through the impedances is $\bar{I}'_a, \bar{I}'_b, \bar{I}'_c$



Study of symmetrical components of a symmetrical three-phase system with a generic triplet of impedances to ground

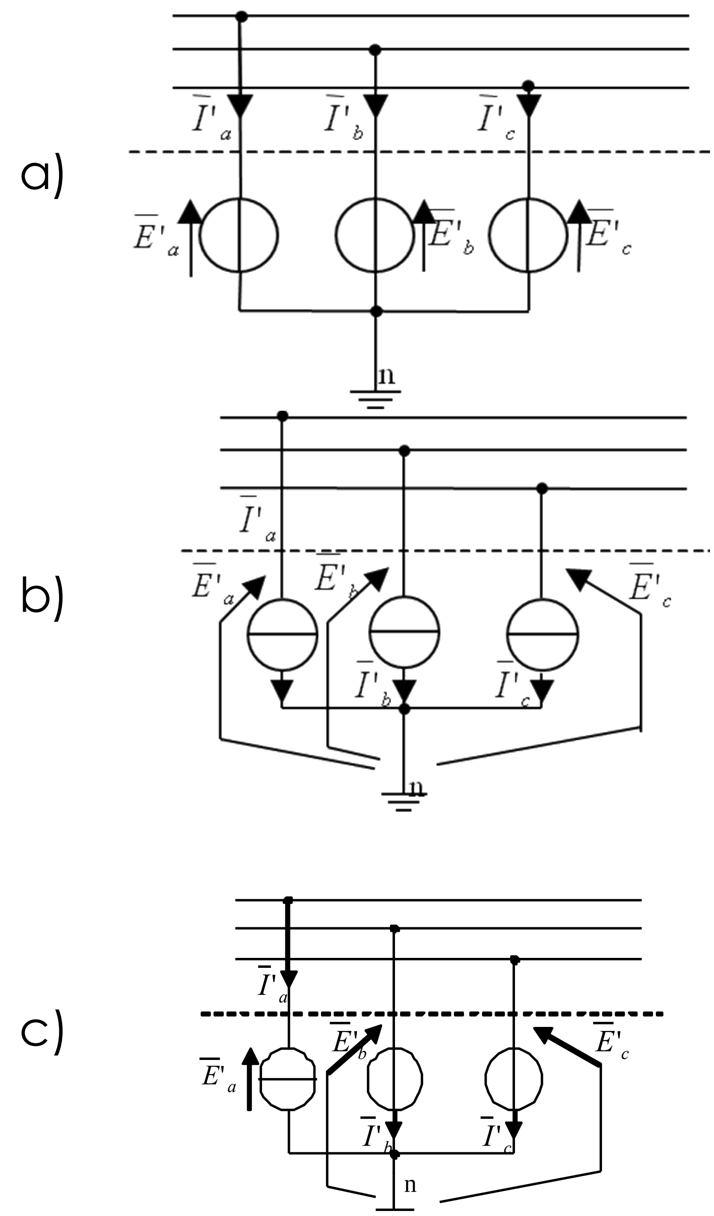
Observation:
We have already shown that **a triplet of generic impedances, in general, cannot be decomposed into three decoupled networks** and therefore the network in the figure must be studied using **symetrically coupled components.**



Study of symmetrical components of a symmetrical three-phase system with a generic triplet of impedances to ground

Observation:

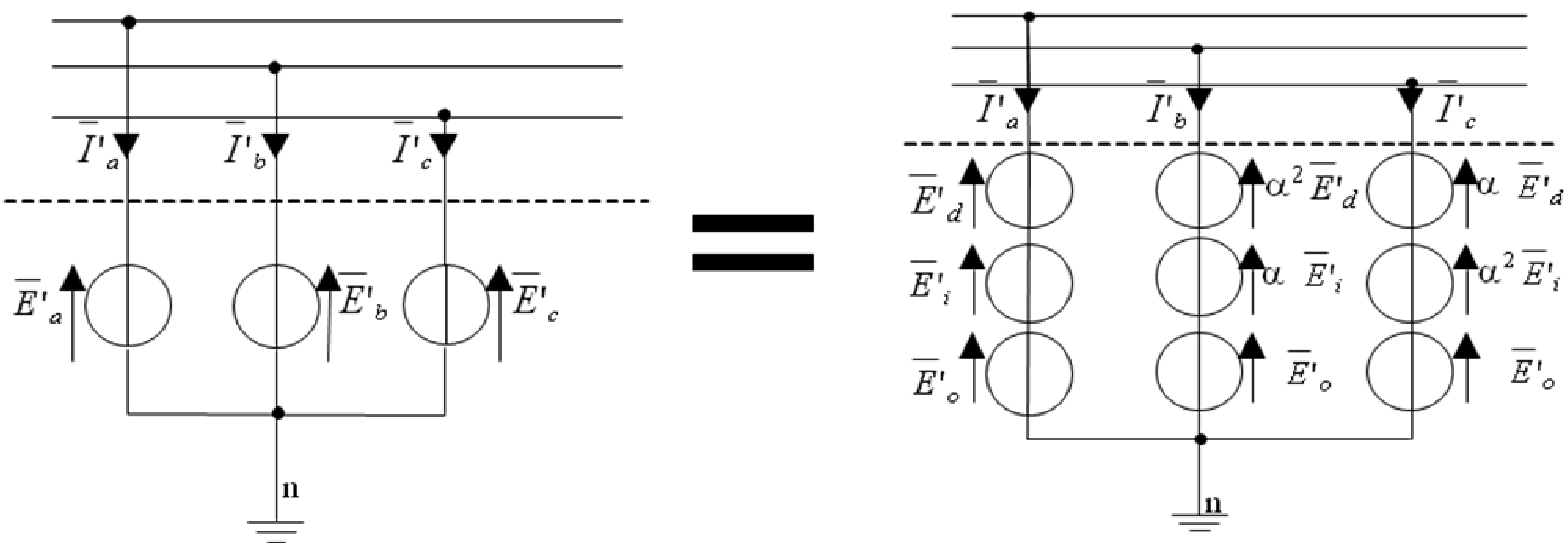
Simplification → connecting the star of impedances shown in the previous figure to a generic network point is **equivalent to inserting a star of voltage sources** $\overline{E}'_a, \overline{E}'_b, \overline{E}'_c$ (figure a) or a star of **current sources** $\overline{I}'_a, \overline{I}'_b, \overline{I}'_c$ (figure b) **or any combination** of voltage and current sources (figure c).



Study of symmetrical components of a symmetrical three-phase system with a generic triplet of impedances to ground

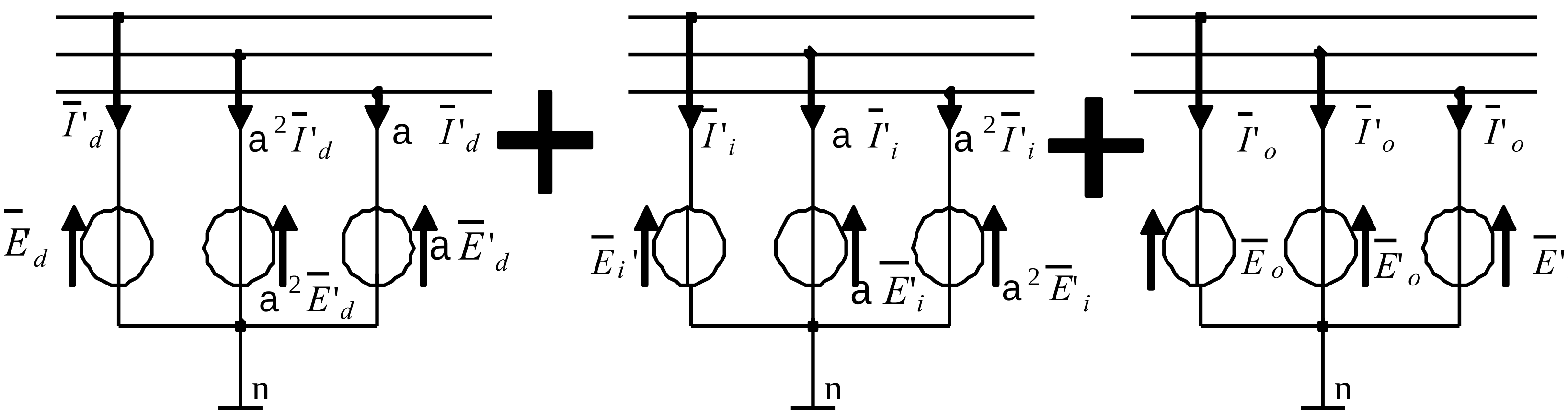
Observation:
Thanks to this assumption, the star of the voltage generators in a) can be decomposed by means of the matrix, i.e., by the relation:

$$\begin{bmatrix} \overline{E}'_a \\ \overline{E}'_b \\ \overline{E}'_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \overline{E}'_0 \\ \overline{E}'_d \\ \overline{E}'_i \end{bmatrix}$$



Study of symmetrical components of a symmetrical three-phase system with a generic triplet of impedances to ground

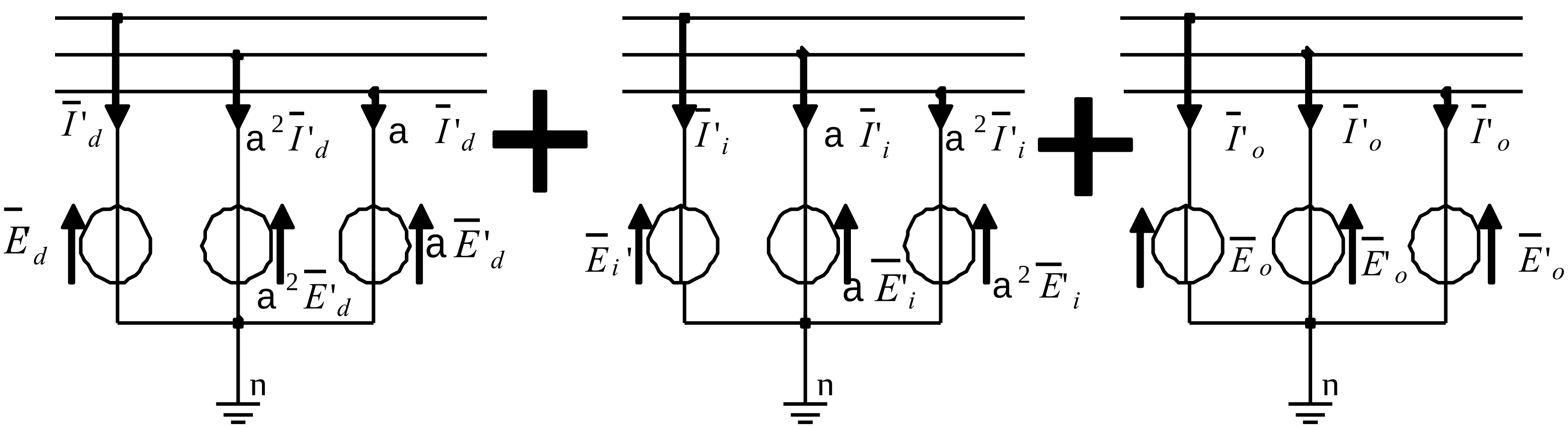
If we apply the **principle of superposition**, **the circuit in the previous figure can be decomposed into the symmetrical components shown below**. So the initial asymmetrical three-phase network has been decomposed into three symmetrical three-phase networks.



Study of symmetrical components of a symmetrical three-phase system with a generic triplet of impedances to ground

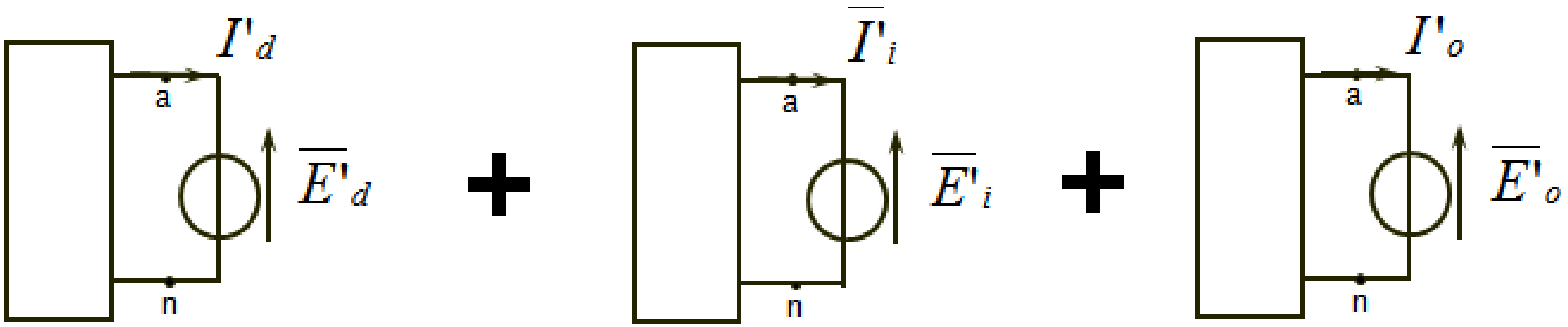
Remarks:

- as long as the starting network is symmetrical, its internal electromotive forces comprise a single sequence, which is the direct sequence;
- The **direct, inverse, and homopolar** currents circulate in the three respective circuits in the figure;
- the three networks in the figure are **symmetrical** and therefore **can be studied as monophase networks**;
- as soon as the various quantities with symmetrical components are calculated, the currents and voltages of the original system are determined using the inverse transformation.

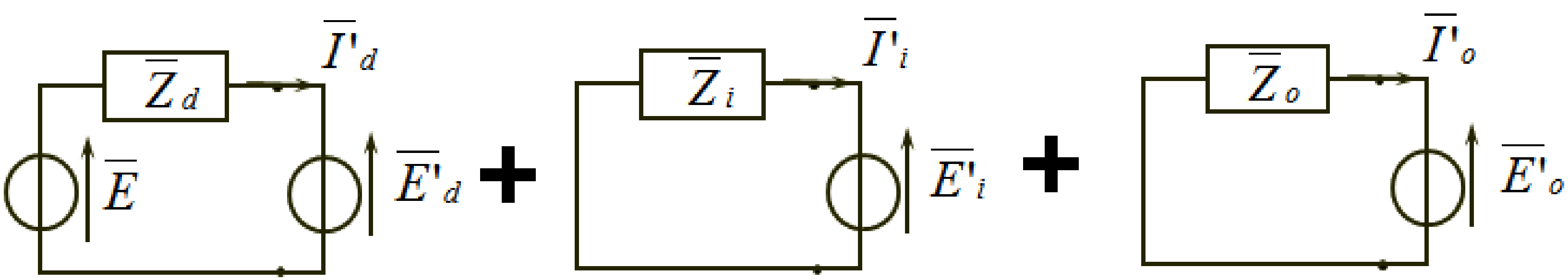


Study of symmetrical components of a symmetrical three-phase system with a generic triplet of impedances to ground

Therefore, we can reduce the three-phase networks shown in the previous figure to three single-phase networks.



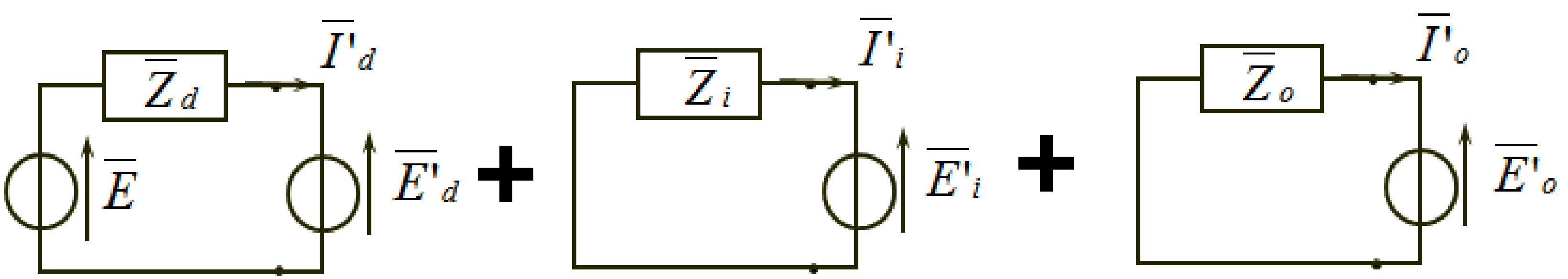
If we use the **Thévenin theorem**, the three networks between points *a* and *n* can be decomposed to the following circuits:



Study of symmetrical components of a symmetrical three-phase system with a generic triplet of impedances to ground

In these circuits, the term $\overline{E}$ is the **voltage before insertion of the asymmetrical impedances** (i.e., before the short-circuit) at the considered point in the network. **This voltage only exists in the direct sequence circuit.** $\overline{Z}_d, \overline{Z}_i, \overline{Z}_0$ are the **equivalent impedances of the network for each sequence as seen from the considered point.** The three equivalent impedances of the network can be calculated if the schematic of the network is known. We should then compute the 6 unknowns $\overline{E'}_d, \overline{E'}_i, \overline{E'}_o, \overline{I'}_d, \overline{I'}_i, \overline{I'}_o$ knowing the 4 quantities $\overline{Z}_a, \overline{Z}_b, \overline{Z}_c, \overline{Z}_n$

As will be shown in the next sections, it is needed to determine three vectors (6 scalars) according to the characteristics of the short-circuit.



Outline

Study of symmetrical components of a symmetrical three-phase system with a generic triplet of impedances to ground

Three-phase short-circuit

Ungrounded two-phase short-circuit

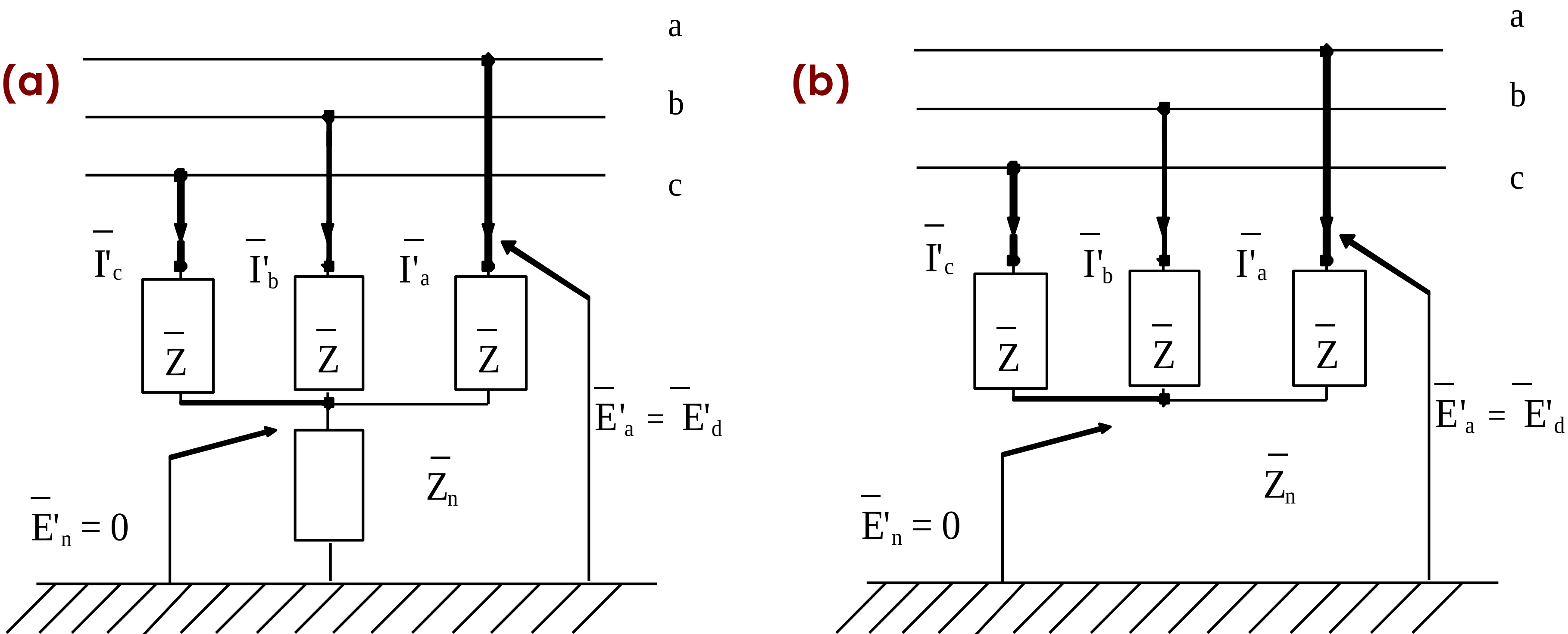
Two-phase short-circuit with connection to ground

Single-phase short-circuit to ground

Three-phase short-circuit

Consider a three-phase short-circuit shown in the figure (a) where, at a **generic point in the network, a star impedance with value $\bar{Z}$ is connected to ground** in series with an impedance $\bar{Z}_n$.
The impedance $\bar{Z}$ is the **impedance of the short-circuit.**

(a)
(b)

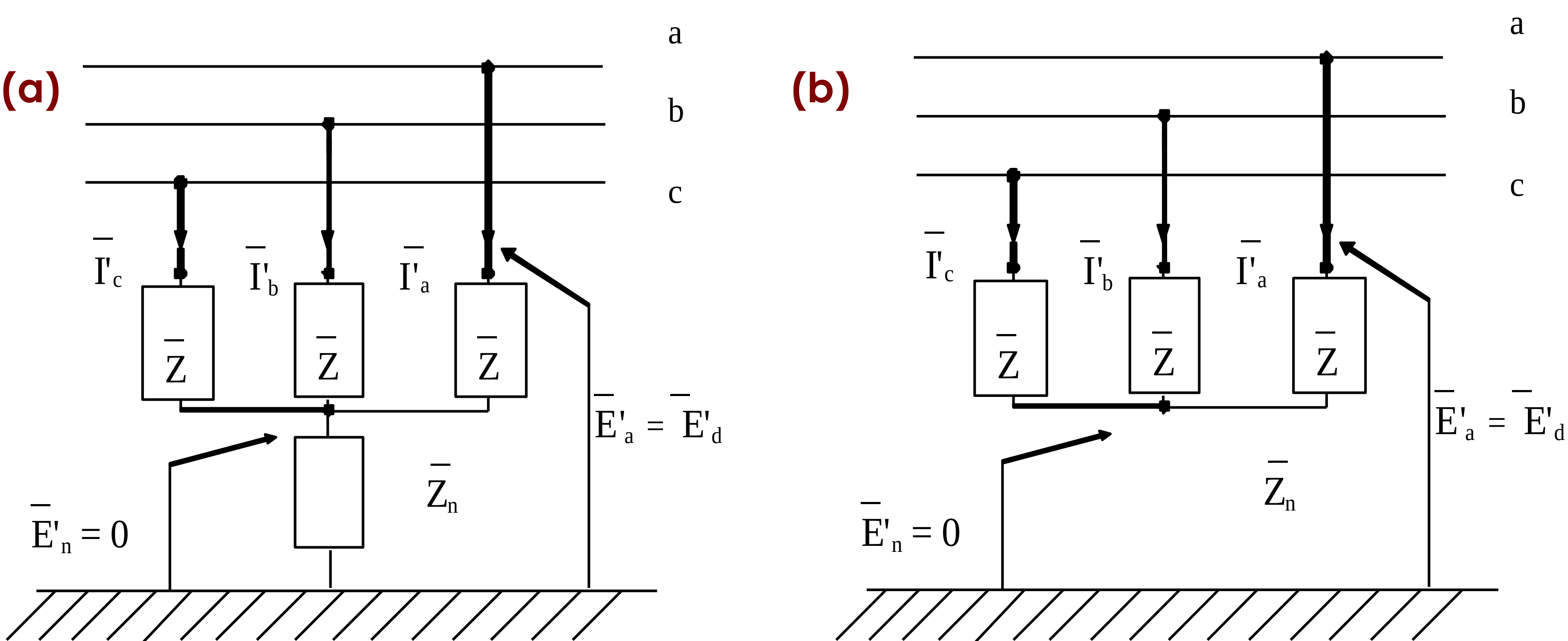


Three-phase short-circuit

Observation: as long as the upstream network is symmetrical, the center of the star of the three impedances $\bar{Z}$ has the same value as the voltage of the star center of the network. Therefore, the impedance $\bar{Z}_n$ can be eliminated.

(a)

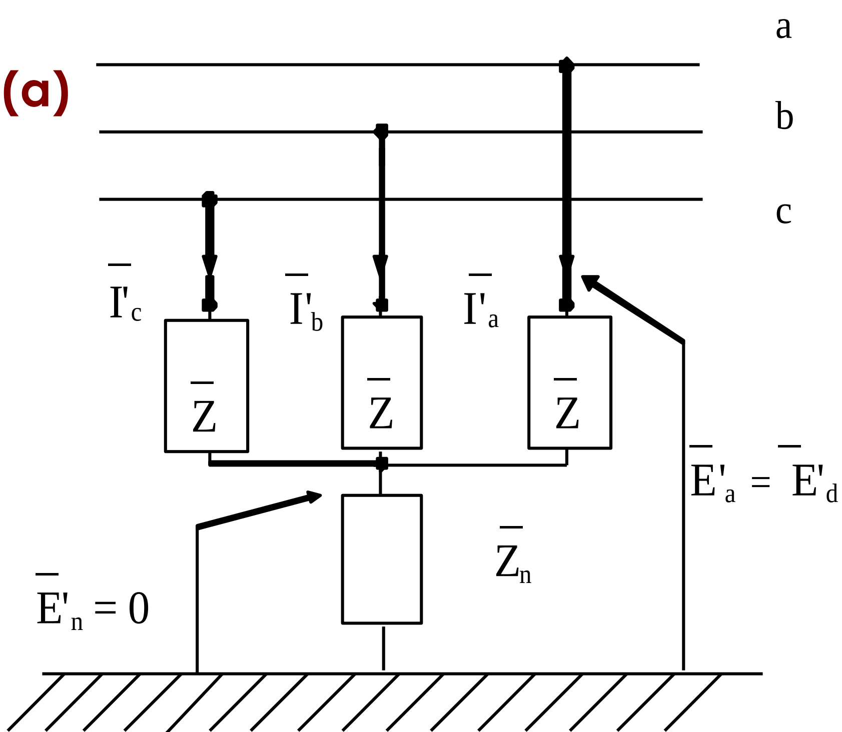
(b)



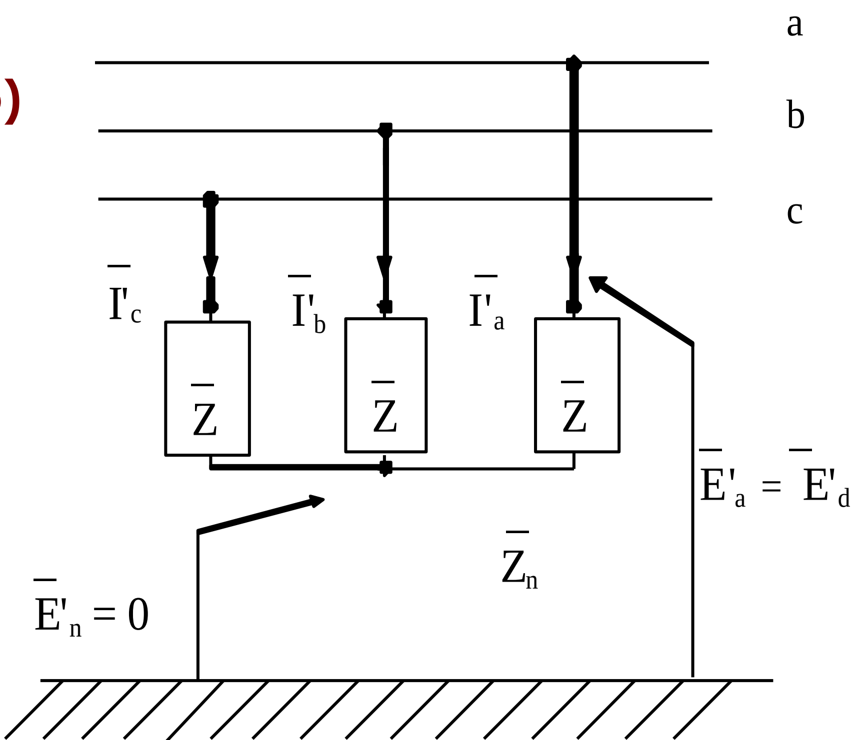
Three-phase short-circuit

Since the considered short-circuit is **symmetrical, the triplet of voltages** $\overline{E}'_a, \overline{E}'_b, \overline{E}'_c$ **and triplet of currents** $\overline{I}'_a, \overline{I}'_b, \overline{I}'_c$ at the point of the short-circuit only has **direct sequence components**, and there are no indirect and homopolar sequence components.

(a)



(b)



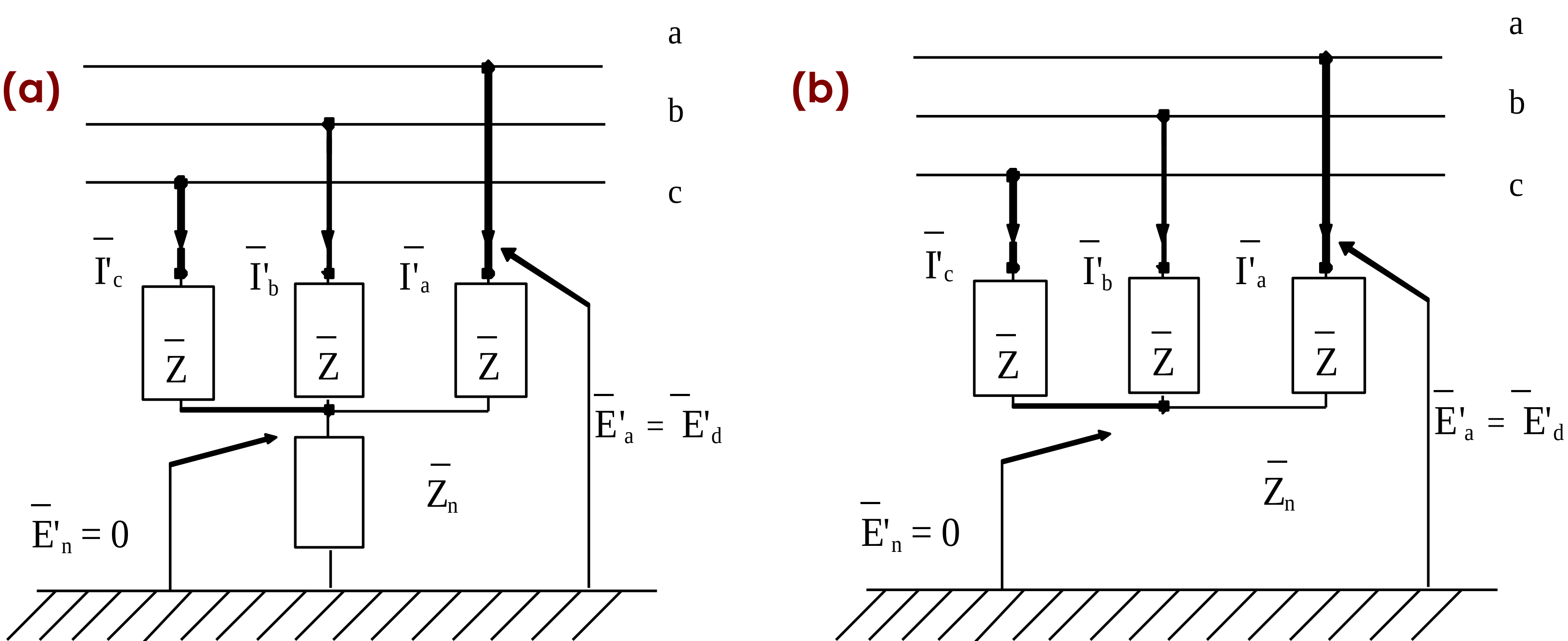
Three-phase short-circuit

The current of the short circuit is, therefore:

$$\overline{I}'_a = \overline{I}'_d = \frac{\overline{E}'_d}{\overline{Z}}$$

For the **direct sequence component:**

$$\overline{E} - \overline{E}'_d = \overline{Z}_d \overline{I}'_d$$



Outline

Study of symmetrical components of a symmetrical three-phase system with a generic triplet of impedances to ground

Three-phase short-circuit

Ungrounded two-phase short-circuit

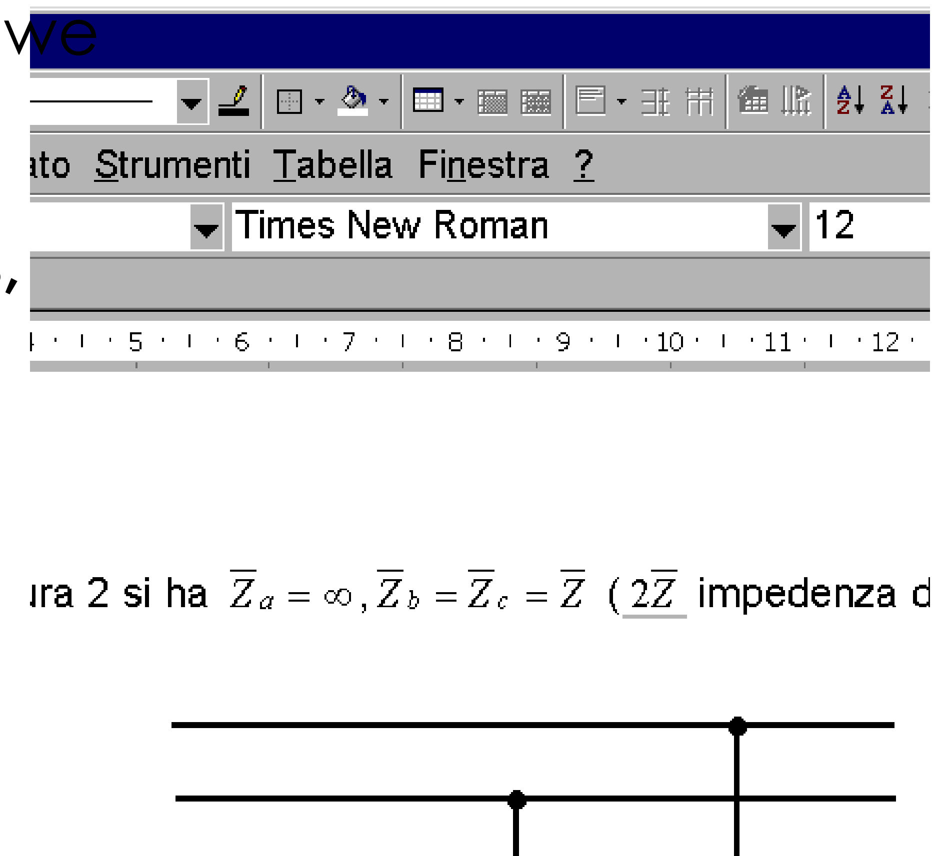
Two-phase short-circuit with connection to ground

Single-phase short-circuit to ground

Ungrounded two-phase short-circuit

We will consider the two-phase short-circuit shown in the figure where, at a generic point in the network, **an impedance is connected between two phases with a short-circuit impedance of** $2\bar{Z}$ **.**

According to Kirchhoff's Law, we can analyze the circuit in the figure to determine **three equations linking the voltages, currents, and short-circuit impedances:**



$$\bar{I}'_a = 0$$

$$\bar{I}'_b + \bar{I}'_c = 0$$

$$\bar{E}'_b - \overline{ZI}'_b = \bar{E}'_c - \overline{ZI}'_c$$

Ungrounded two-phase short-circuit

The **symmetrical component transformation for currents** is introduced, using the first two relations on the preceding slide:

$$\overline{I}_a' = 0$$

$$\overline{I}_b' + \overline{I}_c' = 0$$

$$\begin{bmatrix} \overline{I}_0' \\ \overline{I}_d' \\ \overline{I}_i' \end{bmatrix} = \left[T_s\right]^{-1} \begin{bmatrix} \overline{I}_a' \\ \overline{I}_b' \\ \overline{I}_c' \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} \overline{I}_a' \\ \overline{I}_b' \\ \overline{I}_c' \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ \overline{I}_b' \\ -\overline{I}_b' \end{bmatrix}$$

For this particular case, this relation enables us to obtain the **first two equations in the sequence domain:**

$$\overline{I}_0' = 0$$

$$\overline{I}_d' = -\overline{I}_i' = \frac{\left(a - a^2\right)}{3}\overline{I}_b'$$

Ungrounded two-phase short-circuit

The link between phase currents and symmetrical component currents is as follows:

$$\begin{bmatrix} \bar{I}'_a \\ \bar{I}'_b \\ \bar{I}'_c \end{bmatrix} = \left[T_s\right] \begin{bmatrix} \bar{I}'_0 \\ \bar{I}'_d \\ \bar{I}'_i \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \bar{I}'_0 \\ \bar{I}'_d \\ \bar{I}'_i \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ \bar{I}'_d \\ -\bar{I}'_d \end{bmatrix}$$

Therefore:

$$\bar{I}'_a = 0$$

$$\bar{I}'_b = \left(a^2 - a\right)\bar{I}'_d$$

$$\bar{I}'_c = \left(a - a^2\right)\bar{I}'_d = -\bar{I}'_b$$

Ungrounded two-phase short-circuit

To obtain the **third equation in the sequence domain**, we can use the third equation in the phase domain, specifically:

$$\bar{E}_b' - \overline{ZI}_b' = \bar{E}_c' - \overline{ZI}_c'$$

We can write this equation in matrix form:

$$\bar{E}_b' - \bar{E}_c' = \bar{Z}\left(\bar{I}_b' - \bar{I}_c'\right) = 2\overline{ZI}_b'$$

$$\bar{E}_b' - \bar{E}_c' = \begin{bmatrix} 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \bar{E}_a' \\ \bar{E}_b' \\ \bar{E}_c' \end{bmatrix}$$

Ungrounded two-phase short-circuit

If we use the **sequence transformation for voltages:**

$$\bar{E}_b' - \bar{E}_c' = \begin{bmatrix} 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \bar{E}_a' \\ \bar{E}_b' \\ \bar{E}_c' \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \bar{E}_0' \\ \bar{E}_d' \\ \bar{E}_i' \end{bmatrix} =$$

$$= \begin{bmatrix} 0 & a^2 - a & a - a^2 \end{bmatrix} \begin{bmatrix} \bar{E}_0' \\ \bar{E}_d' \\ \bar{E}_i' \end{bmatrix} = \left(a^2 - a\right)\left(\bar{E}_d' - \bar{E}_i'\right)$$

Therefore, we can write $\bar{E}_b' - \bar{E}_c' = \bar{Z}\left(\bar{I}_b' - \bar{I}_c'\right) = 2\bar{Z}\bar{I}_b'$ as:

$$\left(a^2 - a\right)\left(\bar{E}_d' - \bar{E}_i'\right) = 2\bar{Z}\left(a^2 - a\right)\bar{I}_d' \Rightarrow \left(\bar{E}_d' - \bar{E}_i'\right) = 2\bar{Z}\bar{I}_d'$$

Ungrounded two-phase short-circuit

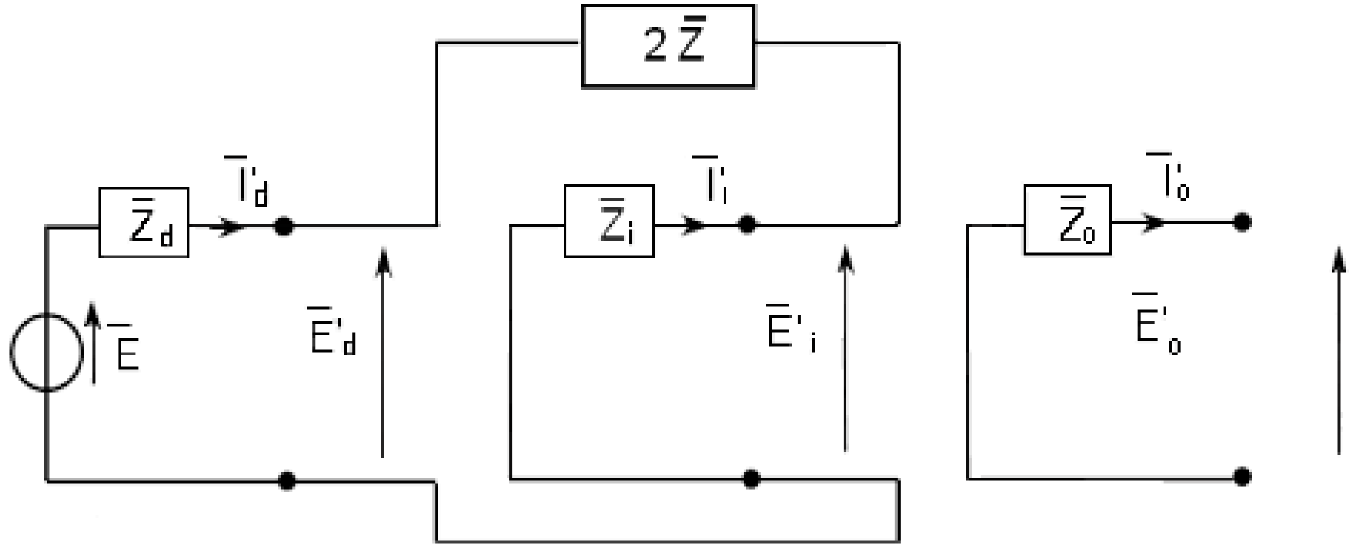
In summary, the necessary equations in the sequence domain are as follows:

$$\overline{I}_0^{'} = 0$$

$$\overline{I}_d^{'} = -\overline{I}_i^{'}$$

$$\left(\overline{E}_d^{'} - \overline{E}_i^{'}\right) = 2\overline{Z}\overline{I}_d^{'}$$

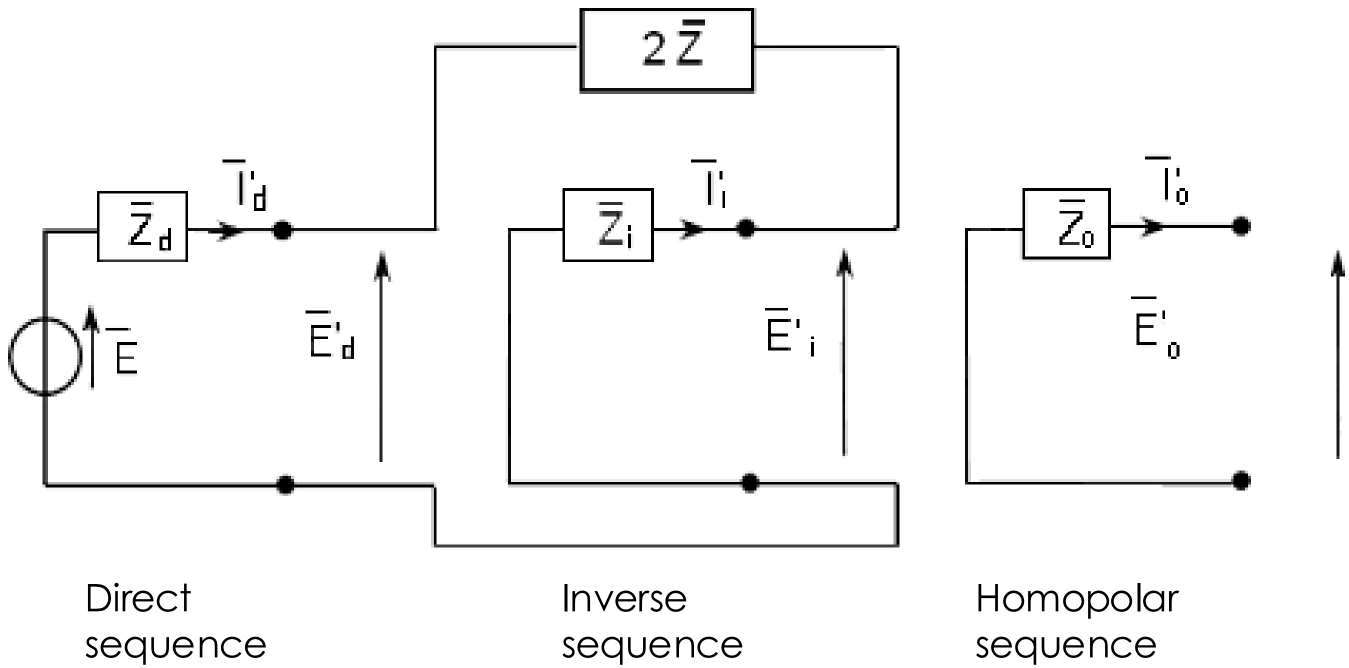
These three equations can be seen as **coupling between sequence circuits.**



Ungrounded two-phase short-circuit

The solution of the circuit enables **determination of the direct and inverse sequence currents** (the homopolar sequence current is zero):

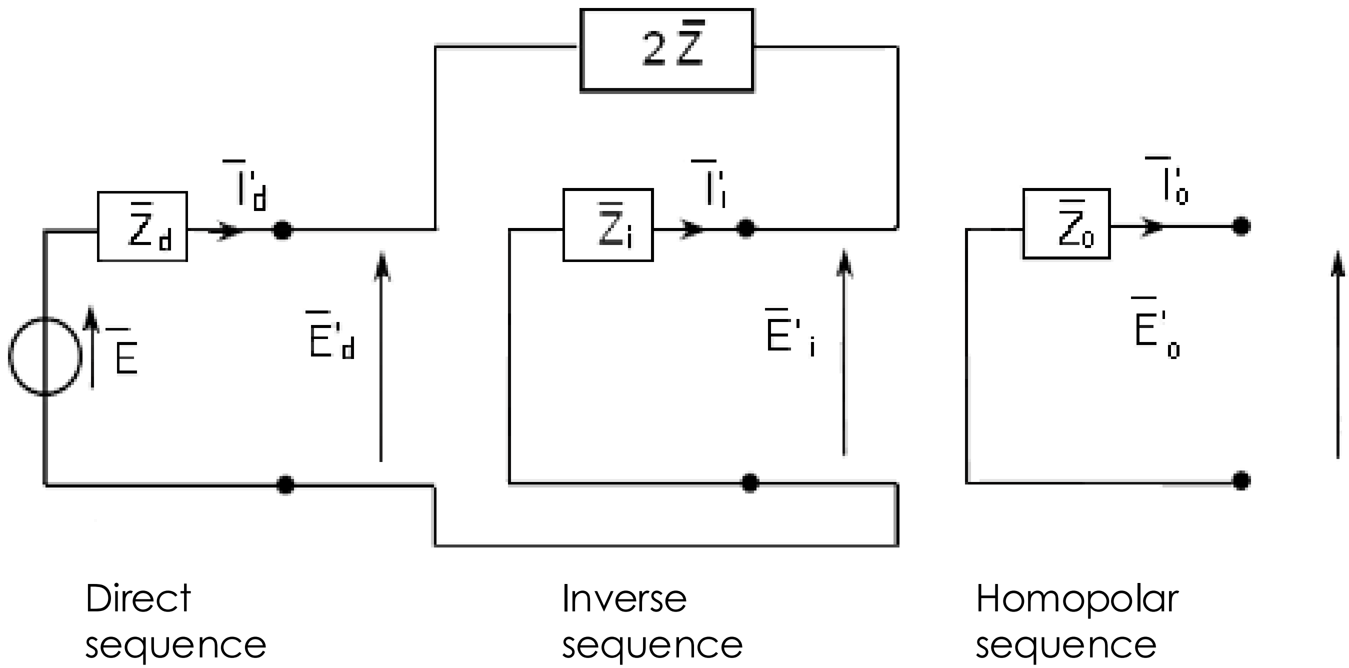
$$\overline{I}_d^{'} = \frac{\overline{E}}{\overline{Z}_d + \overline{Z}_i + 2\overline{Z}} = - \overline{I}_i^{'}$$



Ungrounded two-phase short-circuit

The two **voltages of the direct and inverse sequences** can be determined using the figure:

$$\bar{E}_0^{'} = 0 \qquad \bar{E}_d^{'} = \frac{\bar{E}\left(\bar{Z}_i + 2\bar{Z}\right)}{\bar{Z}_d + \bar{Z}_i + 2\bar{Z}} \qquad \bar{E}_i^{'} = \frac{\bar{E}\bar{Z}_i}{\bar{Z}_d + \bar{Z}_i + 2\bar{Z}}$$

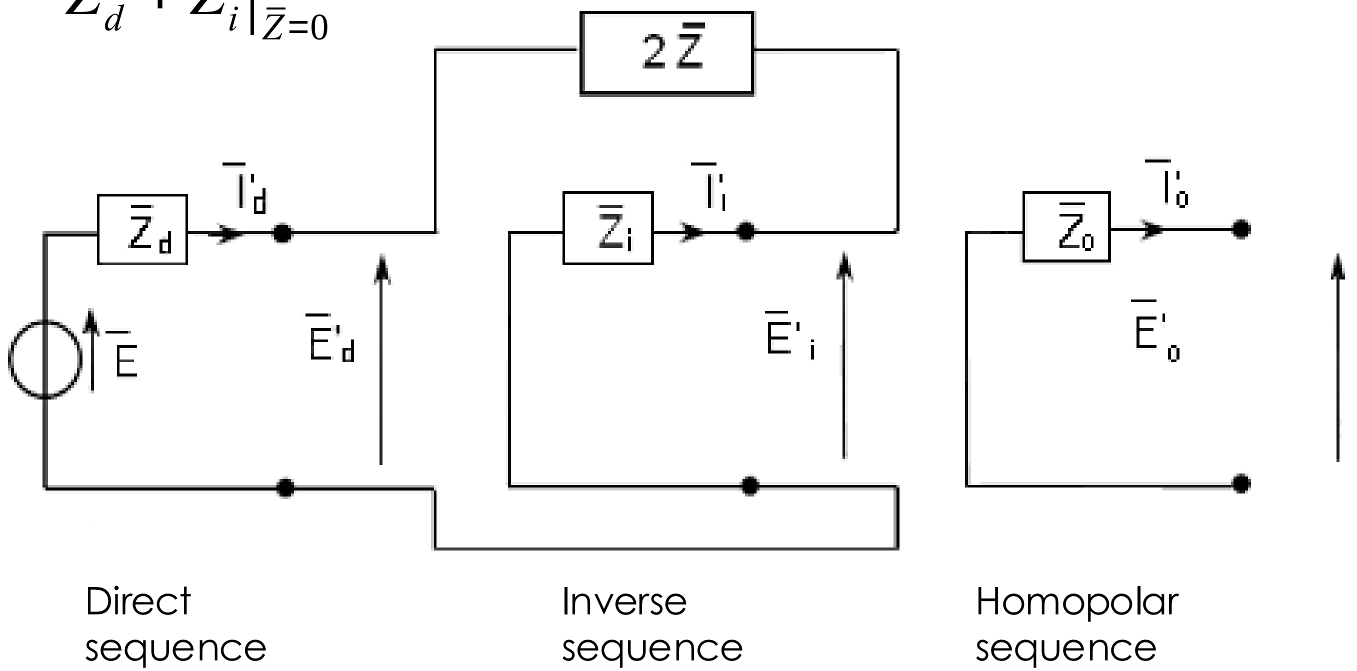


Ungrounded two-phase short-circuit

The **short-circuit current in the phase domain** is given by:

$$\overline{I}_b^{'} = -\left(a - a^2\right)\overline{I}_d^{'} = \left(a^2 - a\right)\frac{\overline{E}}{\overline{Z}_d + \overline{Z}_i + 2\overline{Z}} = -j\sqrt{3}\frac{\overline{E}}{\overline{Z}_d + \overline{Z}_i + 2\overline{Z}} =$$

$$= -j\sqrt{3}\left.\frac{\overline{E}}{\overline{Z}_d + \overline{Z}_i}\right|_{\overline{Z}=0}$$



Outline

Study of symmetrical components of a symmetrical three-phase system with a generic triplet of impedances to ground

Three-phase short-circuit

Ungrounded two-phase short-circuit

Two-phase short-circuit with connection to ground

Single-phase short-circuit to ground

Two-phase short-circuit with connection to ground

We will consider a **two-phase short circuit** shown in the figure where, at a generic point in the network, **an impedance is connected between two phases and the ground with a short-circuit impedance of** $\bar{Z}$ **.**

According to Kirchhoff's Law, we can analyze the circuit in the figure to determine **three equations linking the voltages, currents, and short-circuit impedances:**

$$\bar{I}_a^{'} = 0$$

$$\bar{E}_b^{'} = \bar{E}_c^{'}$$

$$\bar{E}_b^{'} = \bar{Z}\left(\bar{I}_b^{'} + \bar{I}_c^{'}\right)$$

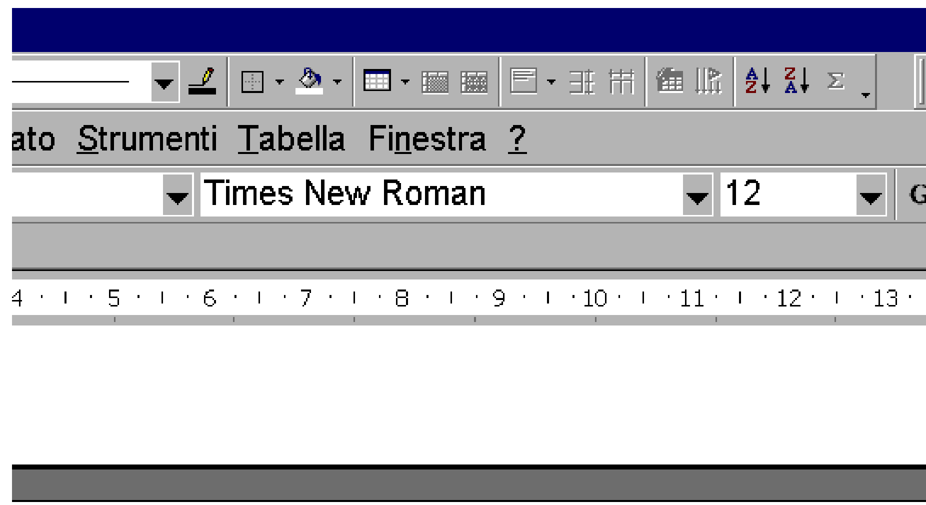


figura 10

Two-phase short-circuit with connection to ground

Using the **sequence transformation for currents** with $\overline{I}_a^{'} = 0$, we obtain:

$\overline{I}_a^{'} = 0$

$$\begin{bmatrix} \overline{I}_a^{'} \\ \overline{I}_b^{'} \\ \overline{I}_c^{'} \end{bmatrix} = \begin{bmatrix} 0 \\ \overline{I}_b^{'} \\ \overline{I}_c^{'} \end{bmatrix} = [T_s] \begin{bmatrix} \overline{I}_0^{'} \\ \overline{I}_d^{'} \\ \overline{I}_i^{'} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \overline{I}_0^{'} \\ \overline{I}_d^{'} \\ \overline{I}_i^{'} \end{bmatrix}$$

Thus:

$$\overline{I}_0^{'} + \overline{I}_d^{'} + \overline{I}_i^{'} = 0$$

Two-phase short-circuit with connection to ground

Using the **inverse sequence transformation for voltages**, we obtain:

$$\begin{bmatrix} \overline{E}_0^{'} \\ \overline{E}_d^{'} \\ \overline{E}_i^{'} \end{bmatrix} = \left[T_s\right]^{-1} \begin{bmatrix} \overline{E}_a^{'} \\ \overline{E}_b^{'} \\ \overline{E}_c^{'} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} \overline{E}_a^{'} \\ \overline{E}_b^{'} \\ \overline{E}_c^{'} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} \overline{E}_a^{'} \\ \overline{E}_b^{'} \\ \overline{E}_b^{'} \end{bmatrix}$$

$$\overline{E}_b^{'} = \overline{E}_c^{'}$$

Therefore:

$$\overline{E}_0^{'} = \frac{1}{3}\left(\overline{E}_a^{'} + 2\overline{E}_b^{'}\right)$$

$$\overline{E}_d^{'} = \frac{1}{3}\left(\overline{E}_a^{'} + \left(a + a^2\right)\overline{E}_b^{'}\right)$$

$$\overline{E}_i^{'} = \frac{1}{3}\left(\overline{E}_a^{'} + \left(a + a^2\right)\overline{E}_b^{'}\right) = \overline{E}_d^{'} \;\text{Þ}\; \overline{E}_i^{'} = \overline{E}_d^{'}$$

Two-phase short-circuit with connection to ground

Recall that:

$$\bar{E}'_b = \bar{Z}\left(\bar{I}'_b + \bar{I}'_c\right)$$

$$\bar{E}_0 = \frac{1}{3}\left(\bar{E}'_a + \bar{E}'_b + \bar{E}'_c\right) \qquad \bar{E}'_a = \left(\bar{E}'_0 + \bar{E}'_d + \bar{E}'_i\right)$$

We obtain:

$$\bar{E}'_0 = \frac{1}{3}\left(\bar{E}'_a + 2\bar{E}'_b\right) \longrightarrow \bar{E}'_0 = \frac{1}{3}\left(\bar{E}'_0 + \bar{E}'_d + \bar{E}'_i + 2\bar{Z}\left(\bar{I}'_b + \bar{I}'_c\right)\right)$$

The term $\left(\bar{I}'_b + \bar{I}'_c\right)$ can be determined using the inverse sequence transformation for currents

$$\begin{bmatrix} \bar{I}'_0 \\ \bar{I}'_d \\ \bar{I}'_i \end{bmatrix} = \left[T_s\right]^{-1} \begin{bmatrix} \bar{I}'_a \\ \bar{I}'_b \\ \bar{I}'_c \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} \bar{I}'_a \\ \bar{I}'_b \\ \bar{I}'_c \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ \bar{I}'_b \\ \bar{I}'_c \end{bmatrix}$$

Two-phase short-circuit with connection to ground

Therefore:

$$\overline{I}'_0 = \frac{\overline{I}'_b + \overline{I}'_c}{3}$$

Finally, by substitution:

$$\overline{E}'_0 = \frac{1}{3}\left(\overline{E}'_0 + \overline{E}'_d + \overline{E}'_i + 6\overline{ZI}'_0\right)$$

If we substitute the following

$$\overline{E}'_i = \frac{1}{3}\left(\overline{E}'_a + \left(a + a^2\right)\overline{E}'_b\right) = \overline{E}'_d \text{ Þ } \overline{E}'_i = \overline{E}'_d$$

in the preceding expression for $\overline{E}'_0$ we obtain:

$$3\overline{E}'_0 = \overline{E}'_0 + 2\overline{E}'_d + 6\overline{ZI}'_0 \text{ Þ } \overline{E}'_0 = \overline{E}'_d + 3\overline{ZI}'_0$$

Two-phase short-circuit with connection to ground

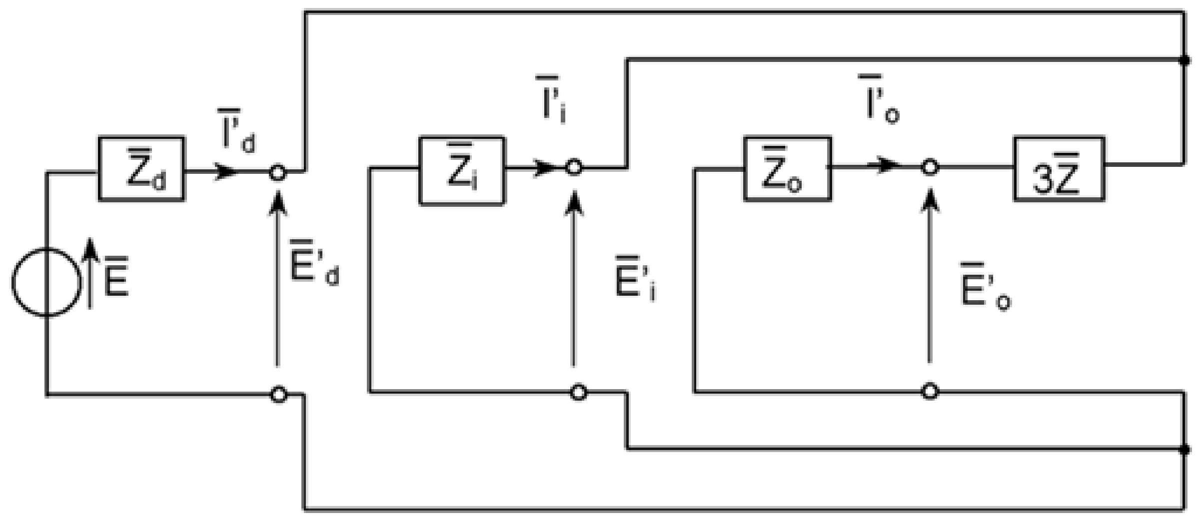
The **three equations of the two-phase grounded short-circuit in the sequence domain** are summarized as follows:

$$\overline{I}_0^{'} + \overline{I}_d^{'} + \overline{I}_i^{'} = 0$$

$$\overline{E}_i^{'} = \overline{E}_d^{'}$$

$$\overline{E}_0^{'} = \overline{E}_d^{'} + 3\overline{Z}\overline{I}_0^{'}$$

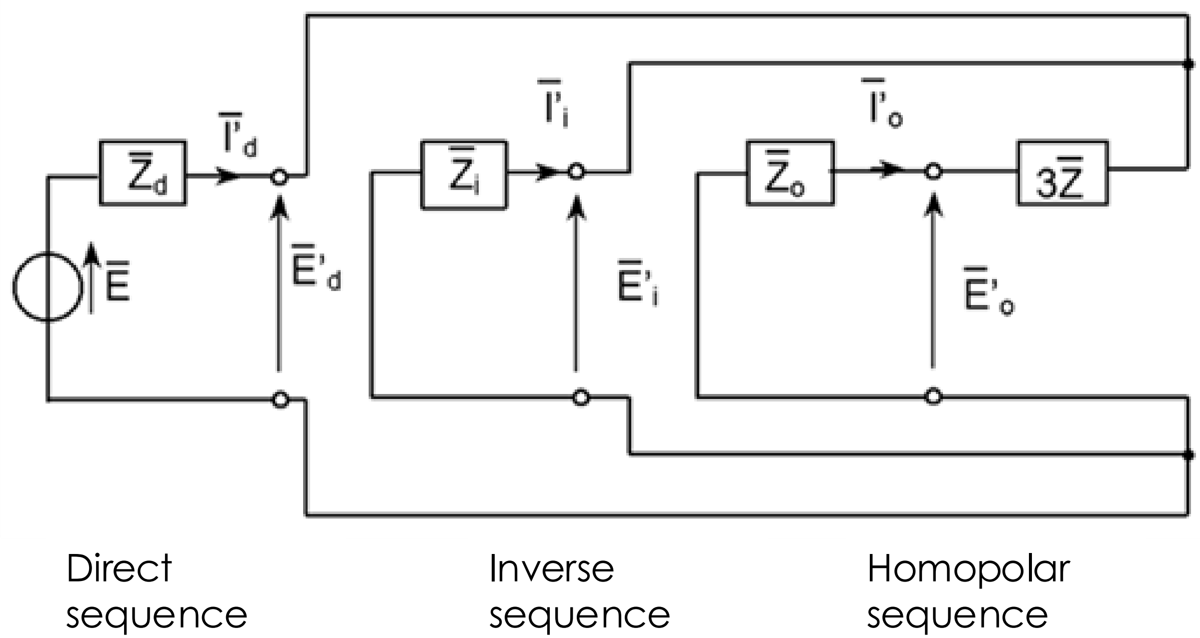
These three equations can be seen as **coupling between sequence circuits.**



Two-phase short-circuit with connection to ground

The circuit solution is as follows:

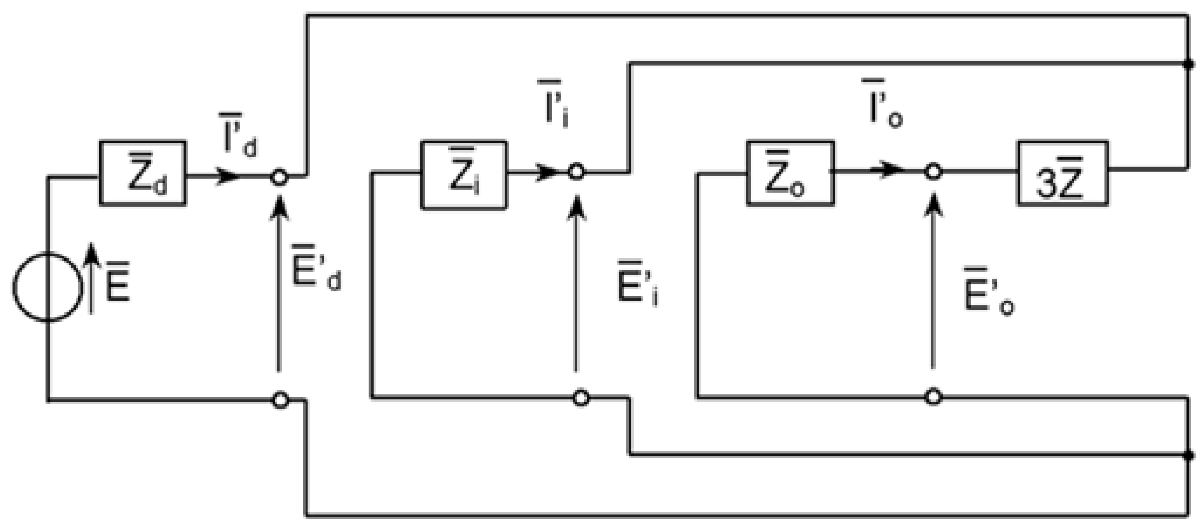
$$\bar{I}_d^{'} = \frac{\bar{E}}{\bar{Z}_d + \dfrac{\bar{Z}_i\left(\bar{Z}_0 + 3\bar{Z}\right)}{\bar{Z}_i + \bar{Z}_0 + 3\bar{Z}}}$$



Two-phase short-circuit with connection to ground

The **inverse and homopolar** currents can be determined if we notice that the impedances $\overline{Z}_0 + 3\overline{Z}$ and $\overline{Z}_i$ are a **current divisor** for $\overline{I}_d^{'}$:

$$\overline{I}_i^{'} = -\,\overline{I}_d^{'} \frac{\overline{Z}_0 + 3\overline{Z}}{\overline{Z}_i + \overline{Z}_0 + 3\overline{Z}} \qquad \overline{I}_0^{'} = -\,\overline{I}_d^{'} \frac{\overline{Z}_i}{\overline{Z}_i + \overline{Z}_0 + 3\overline{Z}}$$

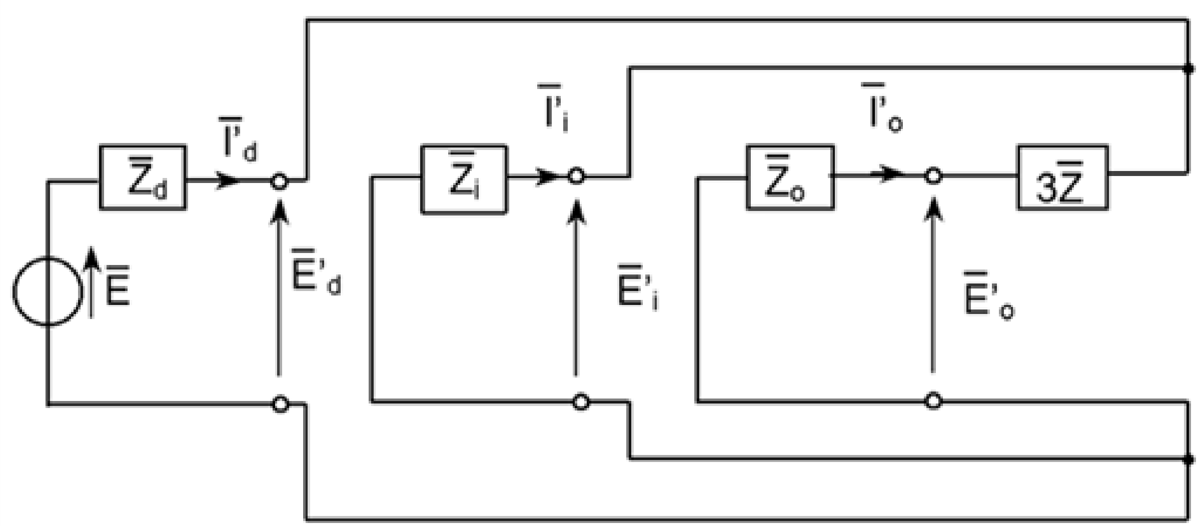


Two-phase short-circuit with connection to ground

The **direct, inverse, and homopolar sequence voltages** are determined using the circuit:

$$\overline{E}_d^{'} = \overline{E} - \overline{Z}_d \overline{I}_d^{'} = \overline{E}_i^{'}$$

$$\overline{E}_0^{'} = - \overline{Z}_0 \overline{I}_0^{'}$$



Direct sequence

Inverse sequence

Homopolar sequence

Two-phase short-circuit with connection to ground

The **short-circuit currents in the phase domain** are:

$$\begin{bmatrix} \bar{I}_a' \\ \bar{I}_b' \\ \bar{I}_c' \end{bmatrix} = \left[T_s\right] \begin{bmatrix} \bar{I}_0' \\ \bar{I}_d' \\ \bar{I}_i' \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -\bar{I}_d' \dfrac{\bar{Z}_i}{\bar{Z}_i + \bar{Z}_0 + 3\bar{Z}} \\ \dfrac{\bar{E}}{\bar{Z}_d + \dfrac{\bar{Z}_i\left(\bar{Z}_0 + 3\bar{Z}\right)}{\bar{Z}_i + \bar{Z}_0 + 3\bar{Z}}} \\ -\bar{I}_d' \dfrac{\bar{Z}_0 + 3\bar{Z}}{\bar{Z}_i + \bar{Z}_0 + 3\bar{Z}} \end{bmatrix}$$

Two-phase short-circuit with connection to ground

Therefore, the phase currents for a **two-phase strongly grounded short-circuit** (i.e., assuming that $\bar{Z}=0$) are as follows:

$$\bar{I}'_a = 0$$

$$\bar{I}'_b = -\bar{I}'_d \frac{\bar{Z}_i}{\bar{Z}_i + \bar{Z}_0 + 3\bar{Z}} + \alpha^2 \bar{I}'_d - \alpha \bar{I}'_d \frac{\bar{Z}_0 + 3\bar{Z}}{\bar{Z}_i + \bar{Z}_0 + 3\bar{Z}} =$$

$$= \bar{I}'_d \left(-\frac{\bar{Z}_i}{\bar{Z}_i + \bar{Z}_0 + 3\bar{Z}} + \alpha^2 - \alpha \frac{\bar{Z}_0 + 3\bar{Z}}{\bar{Z}_i + \bar{Z}_0 + 3\bar{Z}} \right) =$$

$$= \bar{I}'_d \left[\frac{-\bar{Z}_i + \alpha^2(\bar{Z}_i + \bar{Z}_0 + 3\bar{Z}) - \alpha(\bar{Z}_0 + 3\bar{Z})}{\bar{Z}_i + \bar{Z}_0 + 3\bar{Z}} \right] =$$

$$= \bar{I}'_d \left[\frac{\bar{Z}_i(\alpha^2 - 1) + \bar{Z}_0(\alpha^2 - \alpha) + 3\bar{Z}(\alpha^2 - \alpha)}{\bar{Z}_i + \bar{Z}_0 + 3\bar{Z}} \right] =$$

$$= \frac{\bar{E}[\bar{Z}_i(\alpha^2 - 1) + \bar{Z}_0(\alpha^2 - \alpha) + 3\bar{Z}(\alpha^2 - \alpha)]}{\bar{Z}_d(\bar{Z}_i + \bar{Z}_0 + 3\bar{Z}) + \bar{Z}_i(\bar{Z}_0 + 3\bar{Z})}$$

Two-phase short-circuit with connection to ground

Therefore, the phase currents for a **two-phase strongly grounded short-circuit** (i.e., assuming that $\bar{Z}=0$) are as follows:

$$\bar{I}'_c = -\bar{I}'_d \frac{\bar{Z}_i}{\bar{Z}_i+\bar{Z}_0+3\bar{Z}} + \alpha \bar{I}'_d - \alpha^2 \bar{I}'_d \frac{\bar{Z}_0+3\bar{Z}}{\bar{Z}_i+\bar{Z}_0+3\bar{Z}} =$$

$$= \bar{I}'_d \left(-\frac{\bar{Z}_i}{\bar{Z}_i+\bar{Z}_0+3\bar{Z}} + \alpha - \alpha^2 \frac{\bar{Z}_0+3\bar{Z}}{\bar{Z}_i+\bar{Z}_0+3\bar{Z}} \right) =$$

$$= \bar{I}'_d \left[\frac{-\bar{Z}_i + \alpha(\bar{Z}_i+\bar{Z}_0+3\bar{Z}) - \alpha^2(\bar{Z}_0+3\bar{Z})}{\bar{Z}_i+\bar{Z}_0+3\bar{Z}} \right] =$$

$$= \bar{I}'_d \left[\frac{\bar{Z}_i(\alpha-1) + \bar{Z}_0(\alpha-\alpha^2) + 3\bar{Z}(\alpha-\alpha^2)}{\bar{Z}_i+\bar{Z}_0+3\bar{Z}} \right] =$$

$$= \frac{\bar{E}[\bar{Z}_i(\alpha-1) + \bar{Z}_0(\alpha-\alpha^2) + 3\bar{Z}(\alpha-\alpha^2)]}{\bar{Z}_d(\bar{Z}_i+\bar{Z}_0+3\bar{Z}) + \bar{Z}_i(\bar{Z}_0+3\bar{Z})}$$

Outline

Study of symmetrical components of a symmetrical three-phase system with a generic triplet of impedances to ground

Three-phase short-circuit

Ungrounded two-phase short-circuit

Two-phase short-circuit with connection to ground

Single-phase short-circuit to ground

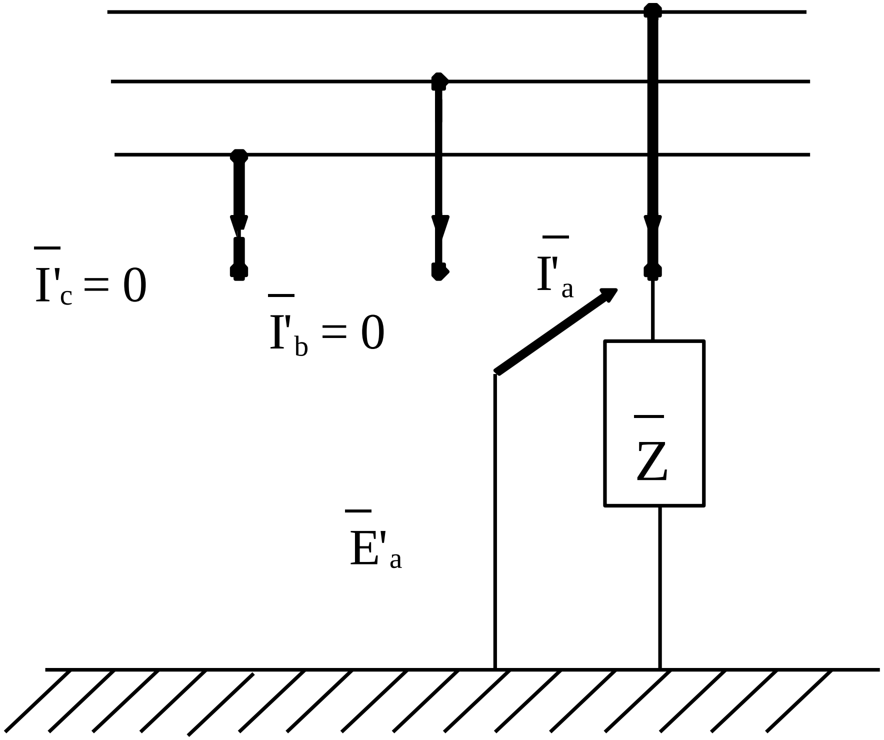
Single-phase short-circuit to ground

The schematic of the short-circuit is shown below.
For this case, the **equations linking voltages, currents, and short-circuit impedance** in the phase domain are:

$$\overline{E}_a' = \overline{Z}\overline{I}_a'$$

$$\overline{I}_b' = 0$$

$$\overline{I}_c' = 0$$



Single-phase short-circuit to ground

We can use the **inverse sequence transformation for currents:**

$$\begin{bmatrix} \overline{I}_0' \\ \overline{I}_d' \\ \overline{I}_i' \end{bmatrix} = \left[T_s\right]^{-1} \begin{bmatrix} \overline{I}_a' \\ \overline{I}_b' \\ \overline{I}_c' \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} \overline{I}_a' \\ \overline{I}_b' \\ \overline{I}_c' \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} \overline{I}_a' \\ 0 \\ 0 \end{bmatrix}$$

$$\overline{I}_b' = 0$$
$$\overline{I}_c' = 0$$

Therefore:

$$\overline{I}_0' = \frac{\overline{I}_a'}{3}$$

$$\overline{I}_d' = \frac{\overline{I}_a'}{3} \Rightarrow \overline{I}_0' = \overline{I}_d' = \overline{I}_i'$$

$$\overline{I}_i' = \frac{\overline{I}_a'}{3}$$

Single-phase short-circuit to ground

The last constraint equation in the sequence domain is determined using the **sequence transformation for voltages:**

$$\begin{bmatrix} \overline{E}'_a \\ \overline{E}'_b \\ \overline{E}'_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \overline{E}'_0 \\ \overline{E}'_d \\ \overline{E}'_i \end{bmatrix}$$

If we make the following substitutions:

$$\overline{I}'_0 = \frac{\overline{I}'_a}{3} \qquad \overline{E}'_a = \overline{ZI}'_a$$

The result is: $\overline{E}'_a = 3\overline{ZI}'_0 = \overline{E}'_0 + \overline{E}'_d + \overline{E}'_i$

Single-phase short-circuit to ground

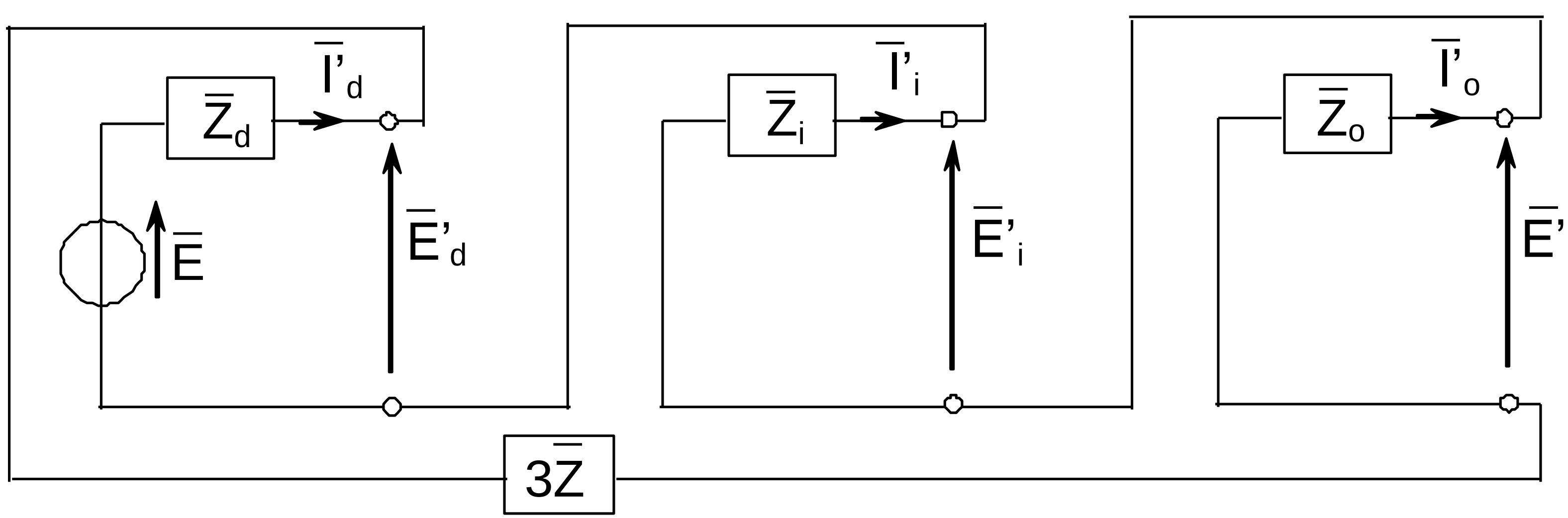
The **three equations in the sequence domain** are summarized as follows:

$$\overline{I}_0' = \overline{I}_d'$$

$$\overline{I}_d' = \overline{I}_i'$$

$$\overline{E}_0' + \overline{E}_d' + \overline{E}_i' = 3\overline{Z}\overline{I}_0'$$

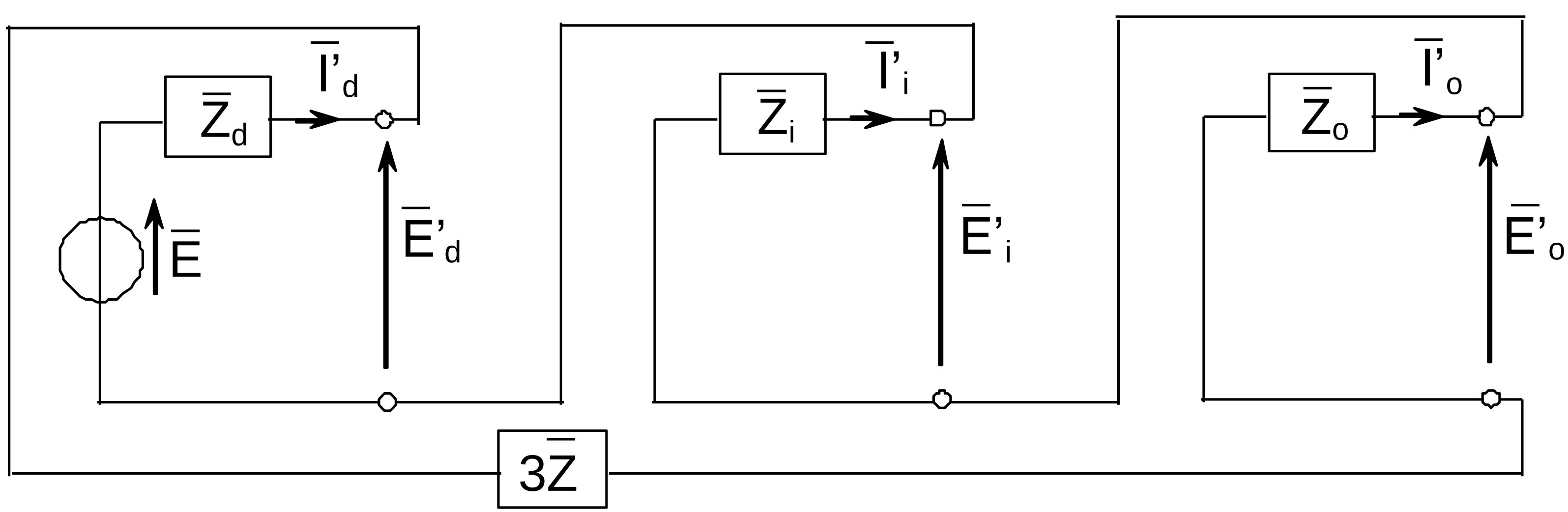
These equations can be seen as a **coupling of the sequence circuits.**



Single-phase short-circuit to ground

The circuit solution enables the determination of the **sequence currents:**

$$\overline{I}_0^{'} = \overline{I}_d^{'} = \overline{I}_i^{'} = \frac{\overline{E}}{\overline{Z}_d + \overline{Z}_i + \overline{Z}_0 + 3\overline{Z}}$$



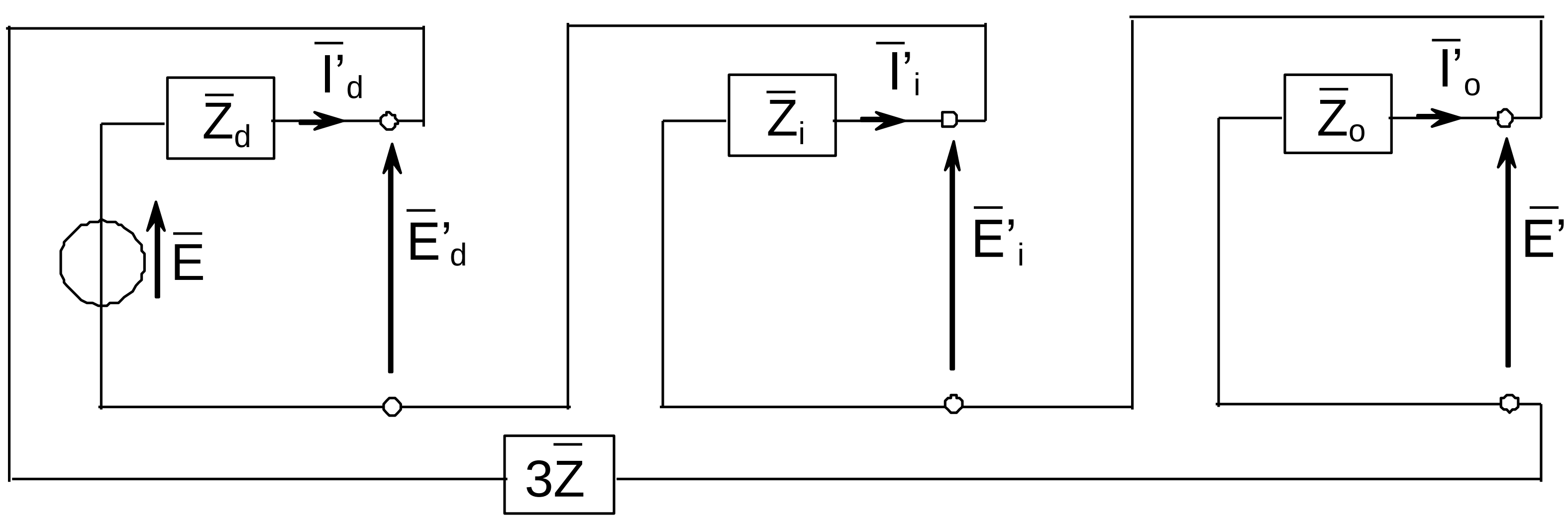
Single-phase short-circuit to ground

The **voltages of each sequence** are:

$$\overline{E}_d^{'} = \overline{E} - \overline{Z}_d \overline{I}_d^{'}$$

$$\overline{E}_i^{'} = - \overline{Z}_i \overline{I}_i^{'}$$

$$\overline{E}_0^{'} = - \overline{Z}_0 \overline{I}_0^{'}$$



Single-phase short-circuit to ground

The **short-circuit current in the phase domain** is as follows:

$$\bar{I}_a' = \frac{3\bar{E}}{\bar{Z}_d + \bar{Z}_i + \bar{Z}_0 + 3\bar{Z}} = \left.\frac{3\bar{E}}{\bar{Z}_d + \bar{Z}_i + \bar{Z}_0}\right|_{\bar{Z}=0}$$